

Let  $A$ ,  $B$ ,  $C$  and  $D$  be 6-bit signed numbers, and let  $P = (A, B)$  and  $Q = (C, D)$  be two points in a two-dimensional rectangular coordinate system. Also, let  $|X|$  be the absolute value of the signed number  $X$ . (In this problem, a signed number is one that is represented using the two's complement number system, and an absolute value is represented as an unsigned number.)

(a) (1 point) In decimal, give the minimum and maximum values for the following quantities:  $A$ ,  $|A|$ ,  $C - A$ ,  $|C - A|$  and  $M = |C - A| + |D - B|$ , where the symbol  $-$  represents subtraction and the symbol  $+$  represents addition. Also, give the minimum number of bits needed to represent each of these quantities without having any overflow. ( $M$  is represented as an unsigned number.)

(b) (1 point) Using only inverters, half adders and 2-to-1 multiplexers, draw the diagram of a circuit having input  $A$  and output  $|A|$ . (You may connect nodes to the constant value 0 or 1, as needed.) The output should have the minimum number of bits as determined in part (a).

(c) (1 point) Using only inverters, half adders, full adders and 2-to-1 multiplexers, draw the diagram of a circuit having inputs  $A$  and  $C$ , and output  $|C - A|$ . (You may connect nodes to the constant value 0 or 1, as needed.) The output should have the minimum number of bits as determined in part (a).

(d) (0.5 points) Using one or more instances of the circuit of part (c) together with full adders, draw the diagram of a circuit having inputs  $A$ ,  $B$ ,  $C$  and  $D$ , and output  $M$ . (You may connect nodes to the constant value 0 or 1, as needed.) The output should have the minimum number of bits as determined in part (a).

(e) (0.5 points) Using only 2-input NAND gates and inverters, draw the diagram for a circuit having the following properties: The inputs are the bits of  $M$ . If  $7 < M < 32$ , then the output is 1; otherwise, the output is 0. Minimize the number of NAND gates used.